

# The Holographic Universe

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## Abstract.

We present a holographic description of four-dimensional single-scalar inflationary universes in terms of a three-dimensional quantum field theory (QFT). The holographic description correctly reproduces standard inflationary predictions in their regime of applicability. In the opposite case, wherein gravity is strongly coupled at early times, we propose a holographic description in terms of perturbative QFT and present models capable of satisfying the current observational constraints while exhibiting a phenomenology distinct from standard inflation. This provides a qualitatively new method for generating a nearly scale-invariant spectrum of primordial cosmological perturbations.

## 1. Introduction.

The notion of holography emerged from black hole physics as an answer to the question: why is the entropy of a black hole proportional to the area of its horizon? Since entropy is an extensive quantity, one would have expected the entropy to be instead proportional to the volume the black hole occupies. Typically, entropy is a measure of the number of degrees of freedom. The scaling of the gravitational entropy has thus been taken as an indication of a new fundamental principle, the holographic principle, that underlies any quantum theory of gravity. More precisely, holography states that any quantum gravitational system in  $(d + 1)$  dimensions should have a dual description in terms of a QFT without gravity in one dimension less [1]. Indeed, if gravity is holographic this would explain the scaling behavior of the black hole entropy, since the entropy of a QFT scales like the volume, which is the same as the area in one dimension higher.

Holography provides a new paradigm for physical reality, the consequences of which we are only just beginning to comprehend. According to this picture, one of the macroscopic dimensions of spacetime and one of the forces in the universe, namely gravity, are emergent phenomena in an underlying lower-dimensional QFT. Concrete realizations of holography have been found in string theory, and a precise holographic dictionary was established shortly thereafter [2, 3, 4]. To date, almost all such realizations involve spacetimes with a negative cosmological constant. The arguments that led to the holographic principle apply more generally, however, and suggest that one should be able to establish a holographic dictionary that applies to our own universe. The purpose of the work presented in [5], and further discussed here, is to propose a concrete holographic framework that applies to our own universe, and in particular to its cosmological evolution. More precisely, we will describe how to set up holography for inflationary cosmology.

Any holographic proposal for cosmology should specify (i) what the dual QFT is, and (ii) how it can be used to compute cosmological observables. Having defined such a duality, the new

description should recover established results in their regime of applicability. Indeed, we will see that our holographic models correctly reproduce standard inflationary results when standard inflation is applicable, namely, under the assumption that gravity was weakly coupled at early times. Yet perhaps more importantly, our holographic approach also gives a qualitatively new method for generating a nearly scale-invariant spectrum when gravity was *strongly* coupled at early times. As we will discuss in the following, there exist holographic models that are capable of satisfying all current observational constraints while exhibiting a phenomenology distinct from standard inflation.

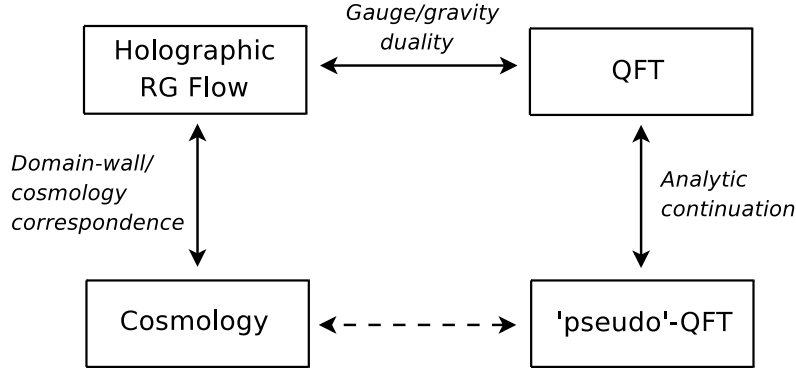
Over the last two decades striking new observations have transformed cosmology from a qualitative to quantitative science [6]. These observations reveal a spatially flat universe, endowed with small-amplitude primordial perturbations that are approximately Gaussian and adiabatic with a nearly scale-invariant spectrum. This data is consistent with the generic predictions of inflationary cosmology and set inflation as the leading theoretical paradigm for the initial conditions of Big Bang cosmology. Yet inflation, despite these successes, is still unsatisfactory in a number of ways: it generically requires fine tuning and there are trans-Planckian issues and questions about the initial conditions for inflation [7].

With future observations promising an unprecedented era of precision cosmology, it becomes imperative that inflation is embedded in a UV complete theory (indeed there is increasing amount of effort devoted to embedding inflation in string theory), and it is also important that alternative scenarios are developed. The holographic approach that we undertake provides both. Firstly, the holographic models we will discuss here are three-dimensional super-renormalizable theories and thus are UV complete. Secondly, holographic dualities are strong/weak coupling dualities meaning that in the regime where one description is weakly coupled, the other is strongly coupled. This provides an arena for constructing new models with intrinsic strong-coupling gravitational dynamics at early times that possess only a weakly coupled three-dimensional QFT description, and are thus outside the class of model described by standard inflation. As we will see, such models may lead to qualitatively different predictions for the cosmological observables that will be measured in the near future. Furthermore, quite apart from the fresh perspective on early universe cosmology such an approach offers, there are also a number of more pragmatic reasons for developing a holographic framework for cosmology: uncovering the structure of three-dimensional QFT in cosmological observables brings in new intuition about their structure and may lead to more efficient computational techniques, *cf.* the computation of non-Gaussianities in [8].

The holographic description we propose uses the one-to-one correspondence between cosmologies and domain-wall spacetimes discussed in [9, 10] and assumes that the standard gauge/gravity duality is valid. More precisely, the steps involved are illustrated in Fig. 1. The first step is to map any given inflationary model to a domain-wall spacetime. For inflationary cosmologies that at late times approach either a de Sitter spacetime or a power-law scaling solution<sup>1</sup>, the corresponding domain-wall solutions describe holographic renormalization group flows. For these cases there is an operational gauge/gravity duality, namely one has a dual description in terms of a three-dimensional QFT. Now, the map between cosmologies and domain-walls may equivalently be expressed entirely in terms of QFT variables, and amounts to a certain analytic continuation of parameters and momenta. Applying this analytic continuation, we obtain the QFT dual of the original cosmological spacetime.

We shall call the resulting theory a ‘pseudo’-QFT because we currently have only an operational definition of this theory. Namely, we do the computations in the QFT dual to the corresponding domain-wall and then apply the analytic continuation. Perhaps a more

<sup>1</sup> This era should then be followed by a hot big bang cosmology, as in standard discussions. Here we only discuss the very early universe, *i.e.*, the times when the primordial cosmological perturbations were generated (the inflationary epoch).



**Figure 1.** The ‘pseudo’-QFT dual to inflationary cosmology is operationally defined using the correspondence of cosmologies to domain-walls and standard gauge/gravity duality.

fundamental perspective is to consider the QFT action, with complex parameters and complex fields as the fundamental objects, and then to consider the results on different real domains as applicable to either domain-walls or cosmologies. Note that the supergravity embedding of the domain-wall/cosmology correspondence discussed in [11] works in precisely this way.

This paper is organized as follows. In the next section we discuss the domain-wall/cosmology correspondence. Then, in Section 3, we discuss the cosmological observables that we would like to compute holographically, and in Section 4 we present the holographic analysis. In Section 5, we discuss the analytic continuation to the pseudo-QFT and in Sections 6 and 7 we discuss the new models that are strongly coupled at early times but have a weakly coupled QFT description.

## 2. Domain-wall/cosmology correspondence.

We explain in this section the lefthand vertical line in Fig. 1, namely, the correspondence between cosmologies and domain-wall spacetimes. For simplicity, we focus on spatially flat universes equipped with a single minimally coupled scalar field, but the results can be extended to more general cases (*eg.*, non-flat, multi-scalar, non-canonical kinetic terms, *etc*). The metric and scalar field for the unperturbed solution are given by

$$ds^2 = \eta dz^2 + a^2(z) d\vec{x}^2, \quad \Phi = \varphi(z), \quad (1)$$

where  $\eta = -1$  in the case of cosmology, in which case  $z$  is the time coordinate, and  $\eta = +1$  in the case of domain-wall solutions<sup>2</sup> in which case  $z$  is the radial coordinate. We take the domain-wall to be Euclidean for later convenience. A Lorentzian domain-wall may be obtained by continuing one of the  $x^i$  coordinates to become the time coordinate [10]. The continuation to a Euclidean domain-wall is convenient, however, because the QFT vacuum state implicit in the Euclidean formulation maps to the Bunch-Davies vacuum on the cosmology side. Other choices of cosmological vacuum require considering the boundary QFT in different states, as may be accomplished using the real-time formalism of [12].

With the appropriate choice of  $a(z)$  and  $\varphi(z)$ , the configuration in (1) solves the field equations that follow from the action

$$S = \frac{\eta}{2\kappa^2} \int d^4x \sqrt{|g|} [-R + (\partial\Phi)^2 + 2\kappa^2 V(\Phi)], \quad (2)$$

where  $\kappa^2 = 8\pi G$  and we are taking the scalar field  $\Phi$  to be dimensionless. Note that when  $\eta = -1$ , the kinetic terms have the appropriate signs for a mostly plus Lorentzian signature

<sup>2</sup> The name ‘domain-wall spacetime’ dates back to earlier work featuring solutions of this form that interpolate between two stationary points of the scalar field potential, one at  $z = +\infty$  and another at  $z = -\infty$ . In the present context the name is somewhat misleading, however, since we consider only the  $z > 0$  part of the geometry. We will nevertheless stick with this terminology as it is standard usage in high-energy physics.

metric and when  $\eta = +1$ , the kinetic terms have the correct sign for Euclidean signature. Had we expressed both actions in the same signature metric, they would differ only in the sign of the potential. It follows that for every flat FRW solution of a model with potential  $V$  there is a corresponding domain-wall solution of a model with potential  $-V$  [9, 10].

For background solutions in which the evolution of the scalar field is (piece-wise) *monotonic*,  $\varphi(z)$  can be inverted to give  $z(\varphi)$  permitting the Hubble rate  $H = \dot{a}/a$  to be re-expressed as  $H(z) = -(1/2)W(\varphi)$ , where  $W(\varphi)$  is known as the ‘fake superpotential’. In this case, the complete equations of motion for the background take the simple form

$$\frac{\dot{a}}{a} = -\frac{1}{2}W, \quad \dot{\varphi} = W_{,\varphi}, \quad 2\eta\kappa^2 V = (W_{,\varphi})^2 - \frac{3}{2}W^2. \quad (3)$$

This first-order formalism goes back to the work of [13] (for cosmology), where it was obtained by application of the Hamilton-Jacobi method. In [10] this formalism was linked to the notion of (fake) (pseudo-) supersymmetry.

We will now extend the correspondence to encompass linear perturbations around the background solution. The linearly perturbed metric takes the general form

$$ds^2 = \eta[1 + 2\phi(z, \vec{x})]dz^2 + 2a^2(z)[\partial_i\nu(z, \vec{x}) + \nu_i(z, \vec{x})]dzdx^i + a^2(z)[\delta_{ij} + h_{ij}(z, \vec{x})]dx^i dx^j, \\ \Phi = \varphi(z) + \delta\varphi(z, \vec{x}), \quad (4)$$

where  $\nu_i$  is transverse. The spatial metric perturbation  $h_{ij}$  may be decomposed as

$$h_{ij}(z, \vec{x}) = -2\psi(z, \vec{x})\delta_{ij} + 2\partial_i\partial_j\chi(z, \vec{x}) + 2\partial_{(i}w_{j)}(z, \vec{x}) + \gamma_{ij}(z, \vec{x}), \quad (5)$$

where  $\omega_i$  is transverse and  $\gamma_{ij}(z, x)$  is transverse traceless. The metric perturbations may then be combined into the gauge-invariant combinations

$$\zeta = \psi + (H/\dot{\varphi})\delta\varphi, \quad (6)$$

$$\hat{\phi} = \phi - (\delta\varphi/\dot{\varphi}), \quad (7)$$

$$\hat{\nu} = \nu - \dot{\chi} - \eta(\delta\varphi/a^2\dot{\varphi}), \quad (8)$$

$$\hat{\nu}_i = \nu_i - \dot{\omega}_i. \quad (9)$$

Physically,  $\zeta$  represents the curvature perturbation on comoving hypersurfaces and has the useful property that it tends to a constant on superhorizon scales<sup>3</sup>.

The equations of motion for cosmological perturbations have been worked out long ago, see [15] and references therein, while the corresponding analysis for domain-walls may be found in [16, 17, 18]. In the present case, there are two independent perturbations represented by  $\zeta$  and  $\gamma_{ij}$ , since the Hamiltonian and momentum constraints are equivalent to

$$\hat{\phi} = -\frac{\dot{\zeta}}{H}, \quad \hat{\nu} = -\frac{\eta\zeta}{a^2H} + \frac{\epsilon\dot{\zeta}}{q^2}, \quad \hat{\nu}_i = 0, \quad (10)$$

where  $\vec{q}$  is the comoving wavevector of the perturbations, and the background quantity  $\epsilon(z)$  is defined as  $\epsilon = -\dot{H}/H^2 = 2(W_{,\varphi}/W)^2$ . (In standard inflation  $\epsilon$  would be the usual slow-roll parameter; however, we do not assume slow roll here). From the remaining Einstein equations, one then finds the following equations of motion for  $\zeta$  and  $\gamma_{ij}$ :

$$0 = \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} - \eta a^{-2}q^2\zeta, \\ 0 = \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} - \eta a^{-2}q^2\gamma_{ij}. \quad (11)$$

<sup>3</sup> Following the end of inflation, the superhorizon value of  $\zeta$  then remains constant until horizon re-entry, irrespective of the dynamics of the intervening evolution, provided that no entropy perturbations are produced [14, 15].

Defining now the analytically continued variables  $\bar{\kappa}$  and  $\bar{q}$  according to

$$\bar{\kappa}^2 = -\kappa^2, \quad \bar{q} = -iq, \quad (12)$$

it is easy to see that a perturbed cosmological solution written in terms of the variables  $\kappa$  and  $q$  continues to a perturbed Euclidean domain-wall solution expressed in terms of the variables  $\bar{\kappa}$  and  $\bar{q}$ . Note that (11) only requires  $\bar{q}^2 = -q^2$  and in (12) we made a choice of a branch cut in the function  $q = \sqrt{q^2}$  (for reasons to be explained in the next section). As it is clear from (3), continuing  $\kappa$  is equivalent to continuing  $V$ . Here we prefer to continue  $\kappa$  since, as we will see, the former has a clear interpretation in terms of dual QFT variables.

We have thus established that the correspondence between cosmologies and domain-walls holds, not only for the background solutions, but also for linear perturbations around them. This is the basis for the relation between power spectra and holographic 2-point functions, to be discussed momentarily. The argument can be generalized to arbitrary order to relate non-Gaussianities to holographic higher-point functions [19].

### 3. Cosmological observables.

In the inflationary paradigm, cosmological perturbations originate on sub-horizon scales as quantum fluctuations of the vacuum. Quantizing the perturbations in the usual manner, one finds the scalar and tensor superhorizon power spectra

$$\begin{aligned} \Delta_S^2(q) &= \frac{q^3}{2\pi^2} \langle \zeta(q) \zeta(-q) \rangle = \frac{q^3}{2\pi^2} |\zeta_{q(0)}|^2, \\ \Delta_T^2(q) &= \frac{q^3}{2\pi^2} \langle \gamma_{ij}(q) \gamma_{ij}(-q) \rangle = \frac{2q^3}{\pi^2} |\gamma_{q(0)}|^2, \end{aligned} \quad (13)$$

where  $\gamma_{q(0)}$  and  $\zeta_{q(0)}$  are the constant late-time values of the cosmological mode functions  $\gamma_q(z)$  and  $\zeta_q(z)$ .

The mode functions are themselves solutions of the classical equations of motion (11) (setting  $\gamma_{ij} = \gamma_q e_{ij}$ , for some time-independent polarization tensor  $e_{ij}$ ). To select a unique solution for each mode function we impose the Bunch-Davies vacuum condition  $\zeta_q, \gamma_q \sim \exp(-iq\tau)$  as  $\tau \rightarrow -\infty$ , where the conformal time  $\tau = \int^z dz'/a(z')$ . The normalization of each solution (up to an overall phase) may then be fixed by imposing the canonical commutation relations for the corresponding quantum fields. This leads to the Wronskian conditions,

$$i = \zeta_q \Pi_q^{(\zeta)*} - \Pi_q^{(\zeta)} \zeta_q^*, \quad i/2 = \gamma_q \Pi_q^{(\gamma)*} - \Pi_q^{(\gamma)} \gamma_q^*, \quad (14)$$

where  $\Pi_q^{(\zeta)} = 2\epsilon a^3 \kappa^{-2} \dot{\zeta}_q$  and  $\Pi_q^{(\gamma)} = (1/4) a^3 \kappa^{-2} \dot{\gamma}_q$  are the canonical momenta associated with each mode function, and we have set  $\hbar$  to unity.

To make connection with the holographic analysis to follow, we introduce the linear response functions  $E$  and  $\Omega$  satisfying

$$\Pi_q^{(\zeta)} = \Omega \zeta_q, \quad \Pi_q^{(\gamma)} = E \gamma_q. \quad (15)$$

These quantities are well-defined since we have already selected a unique solution for each mode function. Substituting these definitions into the Wronskian conditions, which are valid at all times, the cosmological power spectra may be re-expressed as

$$\Delta_S^2(q) = \frac{-q^3}{4\pi^2 \text{Im} \Omega_{(0)}(q)}, \quad \Delta_T^2(q) = \frac{-q^3}{2\pi^2 \text{Im} E_{(0)}(q)}, \quad (16)$$

where  $\text{Im}\Omega_{(0)}$  and  $\text{Im}E_{(0)}$  are the constant late-time values of the imaginary part of the response functions (more precisely, the subscript indicates that this is the part of the response function that is invariant under dilatations, see the discussion at the end of subsection 4.1). We will see shortly how the response functions also give the 2-point function of the pseudo-QFT.

Let us now consider the corresponding domain-wall solution obtained by the applying the continuation (12). The early-time behavior  $\sim \exp(-iq\tau)$  of the cosmological perturbations maps to the exponentially decaying behavior  $\sim \exp(\bar{q}\tau)$  in the interior of the domain-wall ( $\tau \rightarrow -\infty$ ). Such regularity in the interior is a prerequisite for holography, explaining our choice of sign in the continuation of  $q$ .

The domain-wall response functions  $\bar{E}$  and  $\bar{\Omega}$  [18] are defined analogously to (15), namely

$$\bar{\Pi}_{\bar{q}}^{(\zeta)} = -\bar{\Omega} \zeta_{\bar{q}} \quad \bar{\Pi}_{\bar{q}}^{(\gamma)} = -\bar{E} \gamma_{\bar{q}}, \quad (17)$$

where  $\bar{\Pi}_{\bar{q}}^{(\zeta)} = 2\epsilon a^3 \bar{\kappa}^{-2} \dot{\zeta}_{\bar{q}}$  and  $\bar{\Pi}_{\bar{q}}^{(\gamma)} = (1/4)a^3 \bar{\kappa}^{-2} \dot{\gamma}_{\bar{q}}$  are the radial canonical momenta. The minus sign in (17) are inserted so that

$$\bar{\Omega}(-iq) = \Omega(q), \quad \bar{E}(-iq) = E(q). \quad (18)$$

By choosing the arbitrary overall phase of the cosmological perturbations appropriately, we may ensure that the domain-wall perturbations are everywhere real. The domain-wall response functions are then purely real, while their cosmological counterparts are complex.

#### 4. Holographic analysis.

In this section we briefly review relevant material from gauge/gravity duality, corresponding to the upper horizontal line in Fig. 1. There are two classes of domain-wall solutions for which holography is well understood:

*Asymptotically AdS domain-walls.* In this case the solution behaves asymptotically as

$$a(z) \sim e^z, \quad \varphi \sim 0 \quad \text{as } z \rightarrow \infty. \quad (19)$$

The boundary theory has a UV fixed point which corresponds to the bulk AdS critical point. Depending on the rate at which  $\varphi$  approaches zero as  $z \rightarrow \infty$ , the QFT is either a deformation of the conformal field theory (CFT), or else the CFT in a state in which the dual scalar operator acquires a nonvanishing vacuum expectation value (see [20] for details). Under the domain-wall/cosmology correspondence, these solutions are mapped to cosmologies that are asymptotically de Sitter at late times.

*Asymptotically power-law solutions.* In this case the solution behaves asymptotically as

$$a(z) \sim (z/z_0)^n, \quad \varphi \sim \sqrt{2n} \log(z/z_0) \quad \text{as } z \rightarrow \infty, \quad (20)$$

where  $z_0 = n-1$ . This case has only very recently been understood [21]. For  $n = 7$  the asymptotic geometry is the near-horizon limit of a stack of D2 brane solutions. In general, these solutions describe QFTs with a dimensionful coupling constant in the regime where the dimensionality of the coupling constant drives the dynamics. Under the domain-wall/cosmology correspondence, these solutions are mapped to cosmologies that are asymptotically power-law at late times.

#### 4.1. Basics of holography.

Gauge/gravity duality is an exact equivalence between a bulk gravitational theory and a boundary QFT. Typically, the boundary QFT is a gauge theory that admits a large  $N$  expansion. The  $N$  here denotes the rank of the gauge group: an example of such theory, with gauge group  $SU(N)$ , is discussed in section 7.1. The large  $N$  limit consists of taking  $N \rightarrow \infty$  while keeping fixed the 't Hooft coupling constant  $\lambda = g_{YM}^2 N$  [22]. One can show that in this limit only planar diagrams survive. On the bulk side, taking the large  $N$  limit means that one suppresses loop effects. The value of  $\lambda$  then controls whether the supergravity approximation is valid or not.

The duality relates bulk fields with gauge-invariant operators of the boundary theory. For example, the bulk metric corresponds to the boundary stress-energy tensor,  $T_{ij}$ , while bulk scalar fields correspond to boundary scalar operators, such as  $\text{tr } F_{ij} F^{ij}$ , where  $F_{ij}$  is the field strength of the gauge field and the trace is over the gauge group indices. Correlation functions of such gauge invariant operators can then be extracted from the asymptotics of bulk solutions and conversely, given correlation functions of the dual operators, one can reconstruct asymptotic solutions.

To understand the holographic computations we need to know a few things about the structure of asymptotic solutions of the field equations. The discuss below refers specifically to a four dimensional bulk spacetime (the general features are the same in any dimension but the details are different). The results for the case of asymptotically power-law spacetimes can be obtained from the results for asymptotically  $AdS_{2\sigma+1}$  spacetimes via a dimensional reduction on a  $T^{2\sigma-3}$  torus and analytic continuation in  $\sigma$  [23]. The most general asymptotic solution for both cases can be shown to be of the form [24, 21]

$$\begin{aligned} ds^2 &= dr^2 + g_{ij}(r, x) dx^i dx^j, \\ g_{ij}(r, x) &= e^{2r} (g_{(0)ij}(x) + e^{-2r} g_{(2)ij}(x) + \cdots + e^{-2\sigma r} g_{(2\sigma)ij}(x) + \cdots), \end{aligned} \quad (21)$$

where  $\sigma = 3/2$  for solutions of Einstein's equations with negative cosmological constant and  $\sigma = (3n - 1)/2(n - 1) > 3/2$  for the case of asymptotically power-law solutions. In the latter case, the metric above is given in the so-called dual frame [25]: the Einstein frame metric  $g_{ij}^E$  in (2) is related to the metric  $g_{ij}^D$  in (21) via the Weyl transformation  $g_{ij}^E = \exp(\lambda\Phi) g_{ij}^D$ , where  $\lambda = \sqrt{2/n}$ . In (21),  $g_{(0)ij}(x)$  is an arbitrary (non-degenerate) three-dimensional metric, from which the  $g_{(2k)ij}(x)$  with  $k < \sigma$  are locally determined, while  $g_{(2\sigma)ij}(x)$  is only partially constrained by the asymptotic analysis of the field equations. One can show, however, that this coefficient is directly related to the expectation value of the boundary stress-energy tensor [24, 21]:

$$\langle T_{ij} \rangle = \frac{1}{2\bar{\kappa}^2} (2\sigma g_{(2\sigma)ij}). \quad (22)$$

Consider now a scalar field<sup>4</sup>  $\Psi$  that is dual to an operator  $\mathcal{O}$  of dimension  $\Delta$ . The conformal dimension depends on the mass of  $\Psi$  via  $m^2 = \Delta(3 - \Delta)$  in the asymptotically AdS case and is equal to  $\Delta = 4$  in the asymptotically power-law case (for appropriately normalized  $\mathcal{O}$ , see [21]). The asymptotic expansion for  $\Psi$  has a form analogous to (21),

$$\Psi(x, r) = e^{(\Delta-3)r} (\Psi_{(0)} + e^{-2r} \Psi_{(2)} + \cdots + e^{-2\tilde{\sigma}r} \Psi_{(2\tilde{\sigma})}(x) + \cdots), \quad (23)$$

Here  $\Psi_{(0)}(x)$  is unconstrained and is the source for the dual operator  $\mathcal{O}$ , with all subleading terms, up to order  $\exp(-2\tilde{\sigma}r)$ , then being locally determined in terms of the sources. The following term in the series,  $\Psi_{(2\tilde{\sigma})}$ , is undetermined and is related to the expectation value of

<sup>4</sup> In the asymptotically AdS case,  $\Psi = \Phi$ , while in the asymptotically power-law case,  $\Psi = \exp((n-1)\Phi/\sqrt{2n})$ . To match with the discussion in [21] for the D2-brane case ( $n = 7$ ), note that  $\Phi_{\text{here}} = -(\sqrt{14}/5)\phi_{\text{there}}$ , where  $\phi_{\text{there}}$  is the scalar field in [21] for  $p = 2$ .

dual operator  $\langle \mathcal{O} \rangle$ . In the asymptotically AdS case,  $2\tilde{\sigma} = (2\Delta - 3)$ , while in the asymptotically power-law case,  $\tilde{\sigma} = \sigma = (3n - 1)/2(n - 1)$ . (For details, see [20, 21]).

The constraints on  $g_{(2\sigma)ij}$  due to the Einstein equations imply

$$\begin{aligned}\nabla^i \langle T_{ij} \rangle + \langle \mathcal{O} \rangle \nabla_i \Psi_{(0)} &= 0, \\ \langle T_i^i \rangle + (3 - \Delta) \Psi_{(0)} \langle \mathcal{O} \rangle &= 0,\end{aligned}\tag{24}$$

and these are precisely the expected Ward identities<sup>5</sup>.

The relation (21)-(22) can be read in two ways: (i) given a supergravity solution it allows us to read off the QFT data encoded by the solution; (ii) given QFT data it provides a reconstruction of the bulk spacetime in the neighborhood of the boundary. Note that the latter is true even when the supergravity approximation is not valid in the interior of spacetime (because the curvature is large there). The terms exhibited in (21), apart from  $g_{(2\sigma)ij}$ , are non-normalizable terms and are not affected by dynamics. The first term that is affected by dynamics is  $g_{(2\sigma)ij}$  and this is indeed unconstrained asymptotically, except for the constraints due to symmetries (namely the Ward identities (24)). The gauge/gravity duality provides a dual description of dynamics, so the statement of the duality is that  $g_{(2\sigma)ij}$  determined from QFT via (22) should agree with the value obtained from string dynamics in a spacetime with these asymptotics. When the gravity approximation is valid throughout, the asymptotics yield sufficient information to uniquely reconstruct a regular bulk solution: the pair  $(g_{(0)ij}, g_{(2\sigma)ij})$  (and similar for the scalar field) are coordinates in the covariant phase space of the gravitational theory [26].

An alternative way to express these results that we will use in the next subsection is to use the radial Hamiltonian formulation of [18]. This is a Hamiltonian formulation where the radial direction plays the role of time. In this formalism the expectation value of the stress-energy tensor is given by

$$\langle T_{ij} \rangle = \left( \frac{-2}{\sqrt{g}} \bar{\Pi}_{ij} \right)_{(3)}, \tag{25}$$

where  $\bar{\Pi}_{ij}$  is the radial canonical momentum and we will momentarily explain the meaning of subscript (3). There is a similar formula for the expectation value of  $\mathcal{O}$  that we will not need here. A fundamental property of the spacetimes (21) is that the radial derivative is, to leading order as  $r \rightarrow \infty$ , proportional to the dilatation operator  $\delta_D$ ,

$$\partial_r = \delta_D(1 + O[e^{-2r}]), \tag{26}$$

where

$$\delta_D g_{ij}(x, r) = 2g_{ij}(x, r), \quad \delta_D \Psi(x, r) = (\Delta - 3)\Psi(x, r) \tag{27}$$

(one can verify (26) by inspection of (21) and (23)). The expressions in (27) are the standard dilatation transformation rules for a metric and a source that couples to an operator of dimension  $\Delta$  in three spacetime dimensions. Equation (26) is a precise version of the often quoted relation between the radial direction and the energy scale of the dual QFT. In our context, equation (26) implies that one can trade the radial expansion for an expansion in terms of eigenfunctions of dilatation operator. An eigenfunction  $A_{(n)}$  of weight  $n$  is by definition,

$$\delta_D A_{(n)} = -n A_{(n)}. \tag{28}$$

As follows from (26),  $A_{(n)} \sim e^{-nr}(1 + O[e^{-2r}])$ , so the radial expansion and the expansion is eigenfunctions of the dilatation operator are closely related. The chief advantage of the radial

<sup>5</sup> Note there is no conformal anomaly in three dimensions.



Hamiltonian reformulation is that the expansion is now manifestly covariant, whereas expanding in a particular coordinate is not a covariant operation. Now, the radial canonical momentum can be decomposed in eigenfunctions of the dilatation operator and the subscript (3) in (25) indicates that we should pick the part with dilatation eigenvalue<sup>6</sup> 3. One may have expected this on general grounds, since the dimension of stress energy tensor in three dimensions is 3. More generally, the expectation value of an operator of dimension  $\Delta$  is given by the piece of the corresponding radial canonical momentum of weight  $\Delta$ . Note that in general the radial canonical momentum contains also pieces with weight less than  $\Delta$  (*i.e.*, less than 3 in the case of  $T_{ij}$ ): these pieces diverge as  $r \rightarrow \infty$ . One of the advantages of radial Hamiltonian formalism [18] is that renormalization is simple: one simply discards all pieces with weight less than  $\Delta$  (plus additional logarithmic divergences at weight  $\Delta$ , when present). One can indeed show that removing these pieces is equivalent to adding local covariant counterterms to the on-shell action.

The radial canonical momentum for the asymptotically AdS case is equal to

$$\bar{\Pi}_{ij} = \frac{1}{2\bar{\kappa}^2} \sqrt{g} (K_{ij} - K g_{ij}), \quad (29)$$

with  $K_{ij} = (1/2)\partial_r g_{ij}$  the extrinsic curvature of constant- $r$  slices. In the case of asymptotically power-law backgrounds, the relevant canonical momentum is that of the dual frame [21], namely

$$\bar{\Pi}_{ij}^D = \frac{1}{2\bar{\kappa}^2} \sqrt{g} e^{\lambda\Phi} (K_{ij} - (K + \lambda\partial_r\Phi)\delta_j^i), \quad (30)$$

where  $\lambda = \sqrt{2/n}$  and the metric and extrinsic curvature are those of the dual frame.

#### 4.2. 2-point functions.

To linear order in the sources, the variation of the 1-point function for the stress-energy tensor is

$$\delta\langle T_{ij}(x) \rangle = - \int d^3y \sqrt{g_{(0)}} \left( \frac{1}{2} \langle T_{ij}(x) T_{kl}(y) \rangle \delta g_{(0)}^{kl}(y) + \langle T_{ij}(x) \mathcal{O}(y) \rangle \delta\varphi_{(0)}(y) \right). \quad (31)$$

It follows that in order to obtain the holographic 2-point functions we need to solve the linearized equations of motion about the domain-wall solution with Dirichlet boundary conditions at infinity and imposing regularity in the interior. Transforming to momentum space, on general grounds, the 2-point function takes the form

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl}, \quad (32)$$

where  $\Pi_{ijkl}$  is the three-dimensional transverse traceless projection operator defined by

$$\Pi_{ijkl} = \frac{1}{2} (\pi_{ik} \pi_{lj} + \pi_{il} \pi_{kj} - \pi_{ij} \pi_{kl}), \quad \pi_{ij} = \delta_{ij} - \frac{\bar{q}_i \bar{q}_j}{\bar{q}^2}. \quad (33)$$

Decomposing the boundary metric as in (5), we then find

$$\delta\langle T_{ij} \rangle = \frac{1}{2} A(\bar{q}) \gamma_{(0)ij} - 2B(\bar{q}) \psi_{(0)} \pi_{ij} - \langle T_{ij}(\bar{q}) \mathcal{O}(-\bar{q}) \rangle \delta\varphi_{(0)}. \quad (34)$$

From (25), this expression is to be compared with the bulk radial canonical momentum expanded to linear order. We begin by writing the perturbed metric as in (4) and (5), with the lapse and

<sup>6</sup> In odd (bulk) dimensions, the transformation rule of this specific coefficient has also an additional anomalous contribution due to the conformal anomaly. There is no conformal anomaly in our case, and this coefficient is a true eigenfunction of the dilatation operator.

shift perturbations gauged to zero ( $\phi = \nu = \nu_i = 0$ ), and we additionally set  $\omega_i$  to zero using the constraints (10) and a spatial gauge transformation. Then, for the case of asymptotically AdS backgrounds, using (29) we find

$$\delta\langle T_j^i \rangle = \frac{1}{\bar{\kappa}^2} [\delta K \delta_j^i - \delta K_j^i]_{(3)} = -\frac{1}{\bar{\kappa}^2} \left[ 2\dot{\psi} \delta_j^i + \bar{q}^2 \dot{\chi} \pi_j^i + \frac{1}{2} \dot{\gamma}_j^i \right]_{(3)}. \quad (35)$$

Expressing (10) in the present gauge, it follows that

$$\begin{aligned} 2\dot{\psi} &= \dot{\varphi} \delta\varphi, \\ \bar{q}^2 \dot{\chi} &= \frac{\bar{q}^2 \psi}{a^2 H} - \epsilon \dot{\zeta} = \frac{\bar{q}^2 \psi}{a^2 H} + \frac{\bar{\kappa}^2 \bar{\Omega} \zeta}{2a^3} = \left( \frac{\bar{q}^2}{a^2 H} + \frac{\bar{\kappa}^2 \bar{\Omega}}{2a^3} \right) \psi + \frac{\bar{\kappa}^2 \bar{\Omega} H}{2a^3 \dot{\varphi}} \delta\varphi, \end{aligned} \quad (36)$$

where in the last line we have used the definition of the domain-wall response function  $\bar{\Omega}$  in (17) and expanded  $\zeta$  according to (6). Then, using the definition of the response function  $\bar{E}$  also in (17), we find

$$\delta\langle T_j^i \rangle = \left[ \frac{2\bar{E}}{a^3} \gamma_j^i - \left( \frac{\bar{q}^2}{\bar{\kappa}^2 a^2 H} + \frac{\bar{\Omega}}{2a^3} \right) \psi \pi_j^i - \left( \frac{H \bar{\Omega}}{2a^3 \dot{\varphi}} \pi_j^i + \frac{\dot{\varphi}}{\bar{\kappa}^2} \delta_j^i \right) \delta\varphi \right]_{(3)}. \quad (37)$$

Since the scale factor  $a$  has dilatation weight  $-1$  (as follows from (27)), comparison with (34) yields<sup>7</sup>

$$A(\bar{q}) = 4\bar{E}_{(0)}(\bar{q}), \quad B(\bar{q}) = \frac{1}{4} \bar{\Omega}_{(0)}(\bar{q}), \quad (38)$$

where the zero subscript indicates the pieces of the response functions that have zero weight under dilatations, so in particular they are independent of  $r$  as  $r \rightarrow \infty$ . Note that, in general,  $\bar{E}$  and  $\bar{\Omega}$  diverge as  $r \rightarrow \infty$ . Extracting  $\bar{E}_{(0)}$  and  $\bar{\Omega}_{(0)}$  correctly then requires first determining the terms with eigenvalue less than zero and subtracting these from  $\bar{E}$  and  $\bar{\Omega}$ , before taking the limit  $r \rightarrow \infty$  (see [18] and the example in the next subsection). The issue here is that the subtraction of the infinite pieces may induce a change in the finite part as well. This can happen if the local covariant counterterms needed to cancel the infinities necessarily have a finite part as well.

Considering now the case of backgrounds that are asymptotically power-law, as before we decompose the metric perturbations as in (4) and (5), and gauge the shift and vector perturbations to zero ( $\nu = \nu_i = \omega_i = 0$ ). This time, however, we choose the lapse perturbation to be  $\phi = (\lambda/2)\delta\varphi$ . Transforming to the dual frame, we then find

$$\begin{aligned} d\tilde{s}^2 &= e^{-\lambda\Phi} ds^2 = dr^2 + \tilde{a}^2 [\delta_{ij} + \tilde{h}_{ij}] dx^i dx^j, \\ \tilde{h}_{ij} &= -2\tilde{\psi} \delta_{ij} + 2\partial_i \partial_j \tilde{\chi} + \tilde{\gamma}_{ij}, \end{aligned} \quad (39)$$

where  $\tilde{a} = \exp(-\lambda\varphi/2)a$ , with  $\lambda = \sqrt{2/n}$  and the radial coordinate  $dr = \exp(-\lambda\varphi/2)dz$ . The dual frame perturbations are related to their Einstein frame counterparts by

$$\tilde{\psi} = \psi + (\lambda/2)\delta\varphi, \quad \tilde{\chi} = \chi, \quad \tilde{\gamma}_{ij} = \gamma_{ij}. \quad (40)$$

From (25) and (30), we have

$$\begin{aligned} \delta\langle T_j^i \rangle &= \frac{1}{\bar{\kappa}^2} \left[ e^{\lambda\varphi} \left( (\delta\tilde{K} + \lambda\delta\varphi_{,r}) \delta_j^i - \delta\tilde{K}_j^i + (\dots)\delta\varphi \right) \right]_{(3)} \\ &= -\frac{1}{\bar{\kappa}^2} \left[ e^{\lambda\varphi} \left( 2\tilde{\psi}_{,r} \delta_j^i - \lambda\delta\varphi_{,r} \delta_j^i + \bar{q}^2 \tilde{\chi}_{,r} \pi_j^i + \frac{1}{2} \tilde{\gamma}_{j,r}^i + (\dots)\delta\varphi \right) \right]_{(3)} \\ &= -\frac{1}{\bar{\kappa}^2} \left[ e^{3\lambda\varphi/2} \left( 2\dot{\psi} \delta_j^i + \bar{q}^2 \dot{\chi} \pi_j^i + \frac{1}{2} \dot{\gamma}_j^i + (\dots)\delta\varphi \right) \right]_{(3)}. \end{aligned} \quad (41)$$

<sup>7</sup> The term proportional to  $\bar{q}^2$  in the coefficient of  $\psi$  in (37) only contributes a contact term to  $B(\bar{q})$  which we drop.

It follows from (34) that the terms proportional to  $\delta\varphi$  do not contribute to the stress tensor 2-point function that we are interested in, so in the above (and below) these terms have been suppressed. The Einstein frame constraints (10), when expressed in the present gauge, yield

$$2\dot{\psi} = (\dots)\delta\varphi, \quad \bar{q}^2\dot{\chi} = \left(\frac{\bar{q}^2}{a^2H} + \frac{\bar{\kappa}^2\bar{\Omega}}{2a^3}\right)\psi + (\dots)\delta\varphi. \quad (42)$$

Thus we have

$$\delta\langle T_j^i \rangle = \left[ \frac{2\bar{E}}{\tilde{a}^3} \tilde{\gamma}_j^i - \left( \frac{\bar{q}^2 e^{\lambda\varphi/2}}{\bar{\kappa}^2 \tilde{a}^2 H} + \frac{\bar{\Omega}}{2\tilde{a}^3} \right) \tilde{\psi} \pi_j^i + (\dots)\delta\varphi \right]_{(3)}, \quad (43)$$

and since the dilatation weight of  $\tilde{a}$  is  $-1$ , we again recover (38) modulo contact terms.

#### 4.3. Example: Power-law inflation.

To illustrate the above discussion, let us consider the domain-wall backgrounds equal (rather than asymptotic) to (20) discussed in [21], namely

$$a = (z/z_0)^n, \quad \varphi = \sqrt{2n} \ln(z/z_0), \quad z_0 = n - 1 > 0. \quad (44)$$

Under the domain-wall/cosmology correspondence these solutions are mapped to cosmologies undergoing exact power-law inflation. While this model is strongly constrained by the WMAP data [6], this need not concern us here since our purpose is simply to illustrate the steps involved in the holographic computation. Furthermore, in Section 7 we will see that the strong coupling version of these models (*i.e.*, where gravity is strongly coupled at early times but the dual three-dimensional QFT is weakly coupled) are compatible with observations.

Following [10], one can obtain the fake superpotential from the solution yielding  $W = -(2n/z_0) \exp(-\varphi/\sqrt{2n})$ . It follows that  $\epsilon = 1/n$  and both  $\gamma_{ij}$  and  $\zeta$  obey the same equation of motion, which for the domain-wall spacetime reads

$$0 = \ddot{\zeta}_{\bar{q}} + (3n/z)\dot{\zeta}_{\bar{q}} - (z/z_0)^{-2n} \bar{q}^2 \zeta_{\bar{q}}. \quad (45)$$

Imposing regularity in the interior, the solution is

$$\zeta_{\bar{q}} = C_{\bar{q}} \rho^\sigma K_\sigma(\rho), \quad (46)$$

where  $K_\sigma$  is a modified Bessel function of the second kind of order  $\sigma = (3n-1)/2(n-1) > 3/2$ ,  $C_{\bar{q}}$  is an arbitrary function of  $\bar{q}$  and the radial coordinate  $\rho = \bar{q}(z/z_0)^{1-n}$ . The boundary  $z \rightarrow \infty$  corresponds to  $\rho = 0$  while the domain-wall interior corresponds to  $\rho \rightarrow \infty$ . The corresponding radial canonical momentum is equal to

$$\bar{\Pi}_{\bar{q}}^{(\zeta)} = \frac{2\epsilon}{\bar{\kappa}^2} a^3 \dot{\zeta}_{\bar{q}} = -\frac{2C_{\bar{q}}}{n\bar{\kappa}^2} \left( \frac{\rho}{\bar{q}} \right)^{-2\sigma} \rho \partial_\rho (\rho^\sigma K_\sigma(\rho)). \quad (47)$$

Expanding about  $\rho = 0$ , we find

$$\begin{aligned} \zeta_{\bar{q}} &= C_{\bar{q}} \left( 1 + \frac{1}{4(1-\sigma)} \rho^2 + \dots - \frac{\Gamma(1-\sigma)}{4^\sigma \Gamma(1+\sigma)} \rho^{2\sigma} + \dots \right), \\ \bar{\Pi}_{\bar{q}}^{(\zeta)} &= -C_{\bar{q}} \frac{2\bar{q}^{2\sigma}}{n\bar{\kappa}^2} \left( \frac{1}{2(1-\sigma)} \rho^{2(1-\sigma)} + \dots - \frac{2\sigma\Gamma(1-\sigma)}{4^\sigma \Gamma(1+\sigma)} + \dots \right), \end{aligned} \quad (48)$$

and thus

$$\Omega(\bar{q}) = -\frac{\bar{\Pi}_{\bar{q}}^{(\zeta)}}{\zeta_{\bar{q}}} = \frac{2\bar{q}^{2\sigma}}{n\bar{\kappa}^2} \left( \frac{1}{2(1-\sigma)} \rho^{2(1-\sigma)} + \dots - \frac{2\sigma\Gamma(1-\sigma)}{4^\sigma \Gamma(1+\sigma)} + \dots \right). \quad (49)$$

As expected, this diverges as  $\rho \rightarrow 0$ . To compute the 2-point function we need to identify the parts that have negative dilatation eigenvalue, subtract them from (49) and then take  $\rho \rightarrow 0$ .

To do this, we first transform to the dual frame via  $g_{ij}^D = e^{-\sqrt{(2/n)\varphi}} g_{ij}^E$  and then change radial variable,  $r = z_0 \ln(z/z_0) = -\ln(\rho/\bar{q})$ . The metric is now that of AdS,

$$ds^2 = dr^2 + e^{2r} d\vec{x}^2, \quad (50)$$

and the dilatation operator is exactly equal to the radial derivative,

$$\delta_D = \partial_r = -\rho \partial_\rho. \quad (51)$$

(This reflects the fact that the AdS isometry group is the same as the conformal group in one dimension less). It follows that any monomial in  $\rho$  is an eigenfunction of  $\delta_D$ ,

$$\delta_D \rho^n = -n \rho^n, \quad (52)$$

and one can simply identify in (49) all terms with negative eigenvalue; for example,  $\bar{\Omega}_{(-2\sigma+2)} = \bar{q}^{2\sigma}/(n\bar{\kappa}^2(1-\sigma))\rho^{-2(\sigma-1)}$ . We then have

$$\bar{\Omega}_{(0)} = -\frac{4\sigma\Gamma(1-\sigma)}{n4^\sigma\Gamma(1+\sigma)}\bar{\kappa}^{-2}\bar{q}^{2\sigma}. \quad (53)$$

In this example, the identification of the terms with negative eigenvalues could be accomplished by inspection. In more complicated examples, however, this is no longer the case, so we briefly indicate here how one could compute them (see [18] for a more complete discussion). Starting from (45), and using the definition of the corresponding canonical momentum (the first equality in (47)), one obtains

$$\dot{\Pi}_{\bar{q}} - \frac{2\bar{q}^2}{n\bar{\kappa}^2} a \zeta_{\bar{q}} = 0. \quad (54)$$

Inserting in this equation the definition of the response function, and changing the radial coordinate from  $z$  to  $r$ , one obtains

$$\partial_r \bar{\Omega} - \alpha^2 \bar{\Omega}^2 e^{-2\sigma r} + (\bar{q}^2/\alpha^2) e^{2(\sigma-1)r} = 0, \quad (55)$$

where  $\alpha^2 = n\bar{\kappa}^2/2$ . This may now be solved asymptotically by expanding  $\Omega$  in dilatation eigenvalues,

$$\bar{\Omega} = \sum_{k \geq 1} \bar{\Omega}_{(-2\sigma+2k)}, \quad (56)$$

making use<sup>8</sup> of  $\partial_r = \delta_D$  and collecting all terms with the same weight. For example, to leading order, at weight  $(-2\sigma+2)$ , only the first and last term in (55) can have this weight, and one obtains  $\bar{\Omega}_{(-2\sigma+2)} = (\bar{q}^2/\alpha^2 2(1-\sigma)) \exp(2(\sigma-1)r)$  in agreement with (53). Through iteration, one may obtain all coefficients with negative eigenvalue.

Having obtained  $\bar{\Omega}_{(0)}$ , we may finally compute  $B(\bar{q})$ :

$$B(\bar{q}) = \frac{1}{4} \bar{\Omega}_{(0)} = -\frac{\sigma\Gamma(1-\sigma)}{n4^\sigma\Gamma(1+\sigma)}\bar{\kappa}^{-2}\bar{q}^{2\sigma} = -\frac{\pi}{4^\sigma\Gamma^2(\sigma)n\sin\pi\sigma}\bar{\kappa}^{-2}\bar{q}^{2\sigma}. \quad (57)$$

<sup>8</sup> In examples where the background solution is only asymptotically AdS, the relation between the dilatation operator and the radial derivative contains subleading terms (see (26)) and these that must be taken into account in this computation, see [18] for a complete discussion.

A near-identical argument holds for the tensors  $\gamma_{ij}$  yielding  $\bar{\Omega}_{(0)} = (8/n)\bar{E}_{(0)}$ , and hence  $A(\bar{q}) = 2nB(\bar{q})$ . Via the domain-wall/cosmology correspondence, applying the continuations (12) to (53), the imaginary parts of the cosmological response functions are

$$\text{Im}\Omega_{(0)} = (8/n)\text{Im}E_{(0)} = -\frac{4\pi}{n4^\sigma\Gamma^2(\sigma)}\kappa^{-2}q^{2\sigma}. \quad (58)$$

From (16), we then recover the expected cosmological power spectra:

$$\Delta_S^2(q) = \frac{n}{16}\Delta_T^2(q) = \frac{n4^{\sigma-2}\Gamma^2(\sigma)}{\pi^3}\kappa^2q^{3-2\sigma}. \quad (59)$$

Note that we could equally well have obtained (58) by applying the continuations (12) to the *unrenormalized* domain-wall response function (49), and then taking the imaginary part followed by the limit  $z \rightarrow \infty$ . This is because the divergent terms one subtracts to obtain the renormalized response functions are all analytic functions of  $\bar{q}^2$  (as may be seen from (49), where the leading term is proportional to  $\bar{q}^2$ ) and hence under the continuation  $\bar{q}^2 = -q^2$ , these terms remain real and do not contribute to the imaginary part of the cosmological response functions. Only the leading *non-analytic* piece of the domain-wall response functions contributes to the late-time imaginary part of the cosmological response functions: this leading non-analytic piece is finite and is simply  $\bar{\Omega}_{(0)}$ . In fact, the late-time values of the imaginary parts of the cosmological response functions have to be finite as a consequence the Wronskian relations (14) and the fact that  $\zeta$  and  $\gamma_{ij}$  tend to a finite constant at late times.

## 5. Continuation to the pseudo-QFT.

We now wish to re-express the bulk analytic continuation (12) in terms of QFT variables, corresponding to the vertical line on the right-hand side of Fig. 1. First, the dimensionful coupling of the theory, corresponding to the deformation by the operator  $\mathcal{O}$  can be read off from the asymptotics of  $\Phi$ . Since  $\Phi$  does not continue, neither does the coupling constant. Second,  $\bar{\kappa}^{-2}$  is proportional to the square of the number of colors,  $\bar{N}^2$ . It follows that the continuation (12) amounts to

$$\bar{N}^2 = -N^2, \quad \bar{q} = -iq, \quad (60)$$

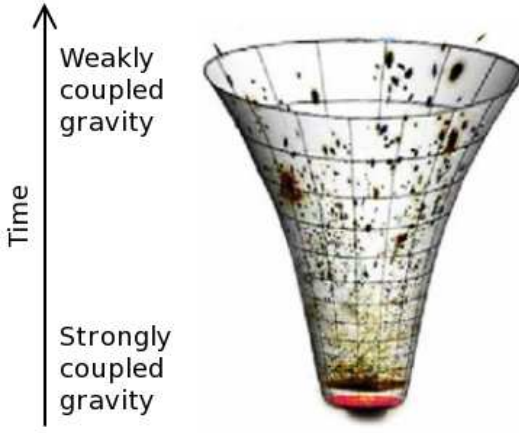
where the barred quantities are associated with the QFT dual to the domain-wall and the unbarred quantities are associated with the pseudo-QFT dual to the cosmology. (Thus  $\bar{N}$  is the rank of the gauge group of the QFT dual to the domain-wall spacetime, while  $N$  is the rank of gauge group of the pseudo-QFT dual to the cosmological spacetime). We therefore find that the power spectrum for any inflationary cosmology that is asymptotically de Sitter or asymptotically power-law can be directly computed from the 2-point function of a three-dimensional QFT via the formulae:

$$\Delta_S^2(q) = \frac{-q^3}{16\pi^2\text{Im}B(-iq)}, \quad \Delta_T^2(q) = \frac{-2q^3}{\pi^2\text{Im}A(-iq)}. \quad (61)$$

This is one of our principal results.

## 6. Beyond the weak gravitational description.

In the discussion so far we have assumed that the description in terms of gravity coupled to a scalar field is valid at early times, and that the perturbative quantization of fluctuations can be justified. The holographic description also allows us to obtain results when these assumptions do not hold. At early times, the theory may be strongly coupled with no useful description in terms of low-energy fields (such as the metric and the scalar field). The holographic set-up allows us to extract the late-time behavior of the system, which can be expressed in terms of



**Figure 2.** Using holography it is possible to describe the generation of primordial cosmological perturbations during an early time epoch in which the gravitational description is *strongly coupled*. At later times we envision a smooth transition to a conventional hot big bang description in which gravity is weakly coupled. (Figure adapted from [27]).

low-energy fields, from QFT correlators. This is the counterpart of the discussion in Section 4 where we saw that in gauge/gravity duality the asymptotic behavior of bulk fields near the boundary of spacetime is reconstructed by the correlators of the dual QFT. This late-time behavior is precisely the information we need to compute the primordial power spectra and other cosmological observables. We will assume that this postulated early-time phase in which gravity is strongly coupled is subsequently followed by a smooth transition to the usual hot big bang cosmology in which gravity is weakly coupled, see Fig. 2.

## 7. Holographic phenomenology for cosmology.

Ideally one would deduce from a string/M-theoretic construction what the dual QFT is. Instead we initiate here a holographic phenomenological approach. The dual QFT would involve scalars, fermions and gauge fields and it should admit a large  $N$  limit. The question is then whether one can find a theory which is compatible with current observations. In particular, one might consider either deformations of CFTs or theories with a single dimensionful parameter in the regime where the dimensionality of the coupling constant drives the dynamics, as these QFTs have already featured in our discussion above.

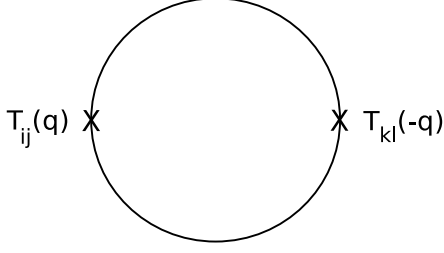
### 7.1. A prototype dual QFT.

We will discuss here super-renormalizable theories that contain one dimensionful coupling constant. A prototype example is three-dimensional  $SU(\bar{N})$  Yang-Mills theory<sup>9</sup> coupled to a number of scalars and fermions, all transforming in the adjoint of  $SU(\bar{N})$ . Theories of this type are typical in AdS/CFT where they appear as the worldvolume theories of D-branes. A general such model that admits a large  $\bar{N}$  limit is

$$S = \frac{1}{g_{\text{YM}}^2} \int d^3x \text{tr} \left[ \frac{1}{2} F_{ij}^I F^{Iij} + \frac{1}{2} (D\phi^J)^2 + \frac{1}{2} (D\chi^K)^2 + \bar{\psi}^L \not{D} \psi^L \right. \\ \left. + \lambda_{M_1 M_2 M_3 M_4} \Phi^{M_1} \Phi^{M_2} \Phi^{M_3} \Phi^{M_4} + \mu_{ML_1 L_2}^{\alpha\beta} \Phi^M \psi_\alpha^{L_1} \psi_\beta^{L_2} \right], \quad (62)$$

where we consider  $\mathcal{N}_A$  gauge fields  $A^I$  ( $I = 1, \dots, \mathcal{N}_A$ );  $\mathcal{N}_\phi$  minimal scalars  $\phi^J$  ( $J = 1, \dots, \mathcal{N}_\phi$ );  $\mathcal{N}_\chi$  conformal scalars  $\chi^K$  ( $K = 1, \dots, \mathcal{N}_\chi$ ) and  $\mathcal{N}_\psi$  fermions  $\psi^L$  ( $L = 1, \dots, \mathcal{N}_\psi$ ). The couplings  $\lambda_{M_1 M_2 M_3 M_4}$  and  $\mu_{ML_1 L_2}^{\alpha\beta}$  (where  $\alpha$  and  $\beta$  are spinor indices) are dimensionless, and we have grouped the scalars appearing in the interaction terms as  $\Phi^M = (\{\phi^J\}, \{\chi^K\})$ . Note

<sup>9</sup> We write the rank of the QFT gauge group here as  $\bar{N}$  since we will first be performing calculations using the QFT dual to the domain-wall spacetime before analytically continuing to the pseudo-QFT.



**Figure 3.** 1-loop contribution to  $\langle T_{ij}(\bar{q})T_{kl}(-\bar{q}) \rangle$ . We sum over the contributions from gauge fields, scalars and fermions, with each diagram yielding a contribution of order  $\sim \bar{N}^2 \bar{q}^3$ .

that all terms in the Lagrangian have dimension 4 so this QFT is indeed of the same type as the QFTs dual to asymptotically power law solutions. The operator  $\mathcal{O}$  that featured in our earlier discussion of holography is closely related to the Lagrangian (see the discussion in Section 4 of [21]).

The conformally coupled scalars have an  $R\chi^2$  coupling when we couple the theory to gravity; on a flat background this means the conformally coupled scalars have a different stress energy tensor from their minimally coupled counterparts. Specifically, on a flat background, the stress tensor is given by

$$T_{ij} = \frac{1}{g_{\text{YM}}^2} \text{tr} \left[ 2F_{ik}^I F_j^{Ik} + D_i \phi^J D_j \phi^J + D_i \chi^K D_j \chi^K - \frac{1}{8} D_i D_j (\chi^K)^2 + \frac{1}{2} \bar{\psi}^L \gamma_{(i} \overleftrightarrow{D}_{j)} \psi^L \right. \\ \left. - \delta_{ij} \left( \frac{1}{2} F_{kl}^I F^{Ikl} + \frac{1}{2} (D\phi^J)^2 + \frac{1}{2} (D\chi^K)^2 - \frac{1}{8} D^2 (\chi^K)^2 \right. \right. \\ \left. \left. + \lambda_{M_1 M_2 M_3 M_4} \Phi^{M_1} \Phi^{M_2} \Phi^{M_3} \Phi^{M_4} + \mu_{ML_1 L_2}^{\alpha\beta} \Phi^M \psi_{\alpha}^{L_1} \psi_{\beta}^{L_2} \right) \right]. \quad (63)$$

To extract predictions, we need to compute the coefficients  $A(\bar{q})$  and  $B(\bar{q})$  appearing in the general decomposition (32) of the stress tensor 2-point function, analytically continue the results, and then insert them in the holographic formulae (61) for the power spectra. To facilitate comparison with the observational data, we will additionally make use of the standard cosmological parameterizations

$$\Delta_S^2(q) = \Delta_S^2(q_0) \left( \frac{q}{q_0} \right)^{n_{S(q)}-1}, \quad \Delta_T^2(q) = \Delta_T^2(q_0) \left( \frac{q}{q_0} \right)^{n_{T(q)}}, \quad (64)$$

where  $\Delta_{S/T}^2(q_0)$  is the scalar/tensor amplitude at some chosen pivot scale  $q_0$ , and  $n_{S/T}(q)$  is the scalar/tensor spectral tilt.

### 7.2. 1-loop calculation.

The leading contribution to the 2-point function of the stress tensor is at one loop (see Fig. 3). Since the stress tensor has dimension three, and the only dimensionful quantity that can appear to this order is  $\bar{q}$  (1-loop amplitudes are independent of  $g_{\text{YM}}^2$ ), it follows that

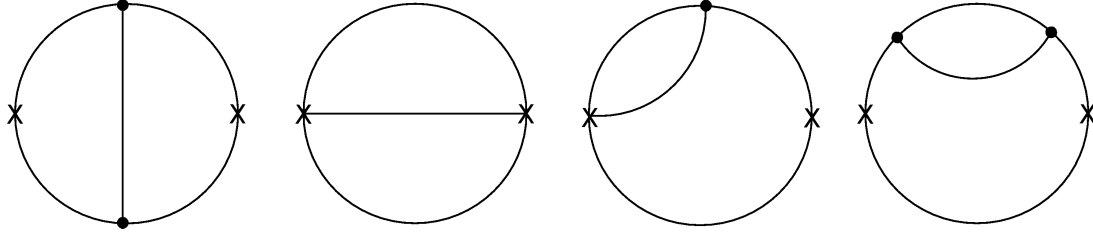
$$A(\bar{q}) = C_A \bar{N}^2 \bar{q}^3 + O(g_{\text{YM}}^2), \quad B(\bar{q}) = C_B \bar{N}^2 \bar{q}^3 + O(g_{\text{YM}}^2), \quad (65)$$

where  $C_A$  and  $C_B$  are numerical coefficients whose value depends only on the field content. Explicit calculation then reveals that

$$C_A = (\mathcal{N}_A + \mathcal{N}_\phi + \mathcal{N}_\chi + 2\mathcal{N}_\psi)/256, \quad C_B = (\mathcal{N}_A + \mathcal{N}_\phi)/256. \quad (66)$$

Inserting (65) into our holographic formulae (61), we find

$$\Delta_S^2(q) = \frac{1}{16\pi^2 \bar{N}^2 C_B} + O(g_{\text{YM}}^2), \quad \Delta_T^2(q) = \frac{2}{\pi^2 \bar{N}^2 C_A} + O(g_{\text{YM}}^2). \quad (67)$$



**Figure 4.** Diagram topologies contributing at 2-loop order. Each diagram consists of an overall factor of  $\bar{N}^3 g_{\text{YM}}^2$  multiplying an integral with superficial degree of divergence two. After dimensional regularization and renormalization, the integrals evaluate to  $\sim \bar{q}^2 \ln(\bar{q}/\bar{q}_*)$ , and so overall each diagram yields a contribution to the stress tensor 2-point function of order  $\sim \bar{N}^3 g_{\text{YM}}^2 \bar{q}^2 \ln(\bar{q}/\bar{q}_*)$ , or equivalently  $\sim \bar{N}^2 \bar{q}^3 g_{\text{eff}}^2 \ln(\bar{q}/\bar{q}_*)$ .

Comparing with (64), we immediately see that the power spectra are *scale-invariant* to leading order (*i.e.*  $n_S = 1 + O(g_{\text{YM}}^2)$ ,  $n_T = O(g_{\text{YM}}^2)$ ), regardless of the precise field content of the model. To estimate the value of  $N$  we may compare with the observed amplitude of the scalar power spectrum. From the WMAP data [6] we have  $\Delta_S^2(q_0) \sim O(10^{-9})$ , hence  $N \sim O(10^4)$ , justifying our use of the large  $N$  limit.

The observational data also serve to provide an upper bound on the ratio of tensor to scalar power spectra. From (67), we find

$$r = \Delta_T^2 / \Delta_S^2 = 32C_B / C_A, \quad (68)$$

and hence an upper bound on  $r$  translates into a constraint on the field content of the dual QFT. A smaller upper bound on  $r$  requires increasing the number of conformal scalars and massless fermions and/or decreasing the number of gauge fields and minimal scalars.

### 7.3. 2-loop corrections.

Corrections to the stress tensor 2-point function at 2-loop order<sup>10</sup> give rise to small deviations from scale invariance. The full result will be reported elsewhere [19], however, it is easy to obtain an order of magnitude estimate on general grounds. The perturbative expansion depends on the effective dimensionless coupling constant  $g_{\text{eff}}^2 = g_{\text{YM}}^2 \bar{N} / \bar{q}$ . Either from inspection or from direct calculation of some of the diagrams contributing at  $O(g_{\text{eff}}^2)$  (see Fig. 4), one finds

$$\begin{aligned} A(\bar{q}) &= C_A \bar{N}^2 \bar{q}^3 [1 + D_A g_{\text{eff}}^2 \ln(\bar{q}/\bar{q}_*) + O(g_{\text{eff}}^4)], \\ B(\bar{q}) &= C_B \bar{N}^2 \bar{q}^3 [1 + D_B g_{\text{eff}}^2 \ln(\bar{q}/\bar{q}_*) + O(g_{\text{eff}}^4)], \end{aligned} \quad (69)$$

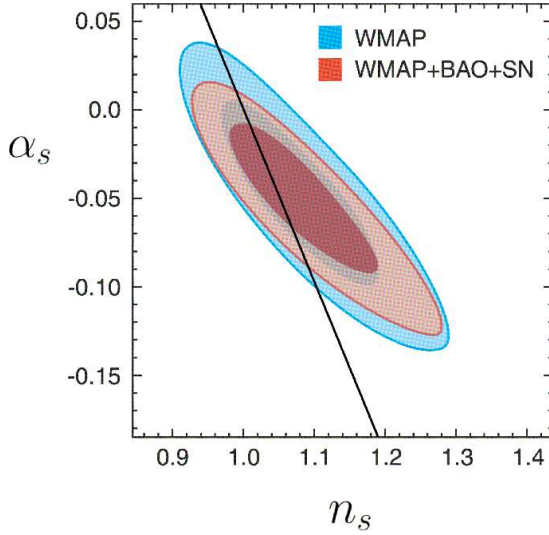
where  $D_A$  and  $D_B$  are numerical coefficients of order one whose value depends only on the field content. To compute  $D_A$  and  $D_B$  precisely requires summing all the relevant 2-loop diagrams.

Inserting these two-loop corrected results into the holographic formulae (61), we find

$$\begin{aligned} \Delta_S^2(q) &= \frac{1}{16\pi^2 \bar{N}^2 C_B} [1 - D_B g_{\text{eff}}^2 \ln(q/q_*) + O(g_{\text{eff}}^4)], \\ \Delta_T^2(q) &= \frac{2}{\pi^2 \bar{N}^2 C_A} [1 - D_A g_{\text{eff}}^2 \ln(q/q_*) + O(g_{\text{eff}}^4)], \end{aligned} \quad (70)$$

<sup>10</sup> Super-renormalizable theories have infrared divergences, but large  $N$  resummation leads to well-defined expressions with  $g_{\text{YM}}^2$  effectively playing the role of an infrared regulator. The exact amplitudes are nonanalytic functions of the coupling constant [28]. Note that our analytic continuation to pseudo-QFT does not involve the coupling constant.





**Figure 5.** The straight line is the leading order prediction of holographic models with a single dimensionful coupling constant for the correlation of the running  $\alpha_s$  and the scalar tilt  $n_s$ . The data show the 68% and 95% CL constraints (marginalizing over tensors) at  $q_0 = 0.002 \text{ Mpc}^{-1}$ , and are taken from Fig. 4 of [6]. As new data appear the allowed region should shrink to a point, which is predicted to lie close to the line.

where the analytically continued effective coupling  $g_{\text{eff}}^2 = g_{\text{YM}}^2 N/q$ . In comparison, expanding (64) yields

$$\begin{aligned}\Delta_S^2(q) &= \Delta_S^2(q_0)[1 + (n_S(q)-1)\ln(q/q_0) + O((n_S(q)-1)^2)], \\ \Delta_T^2(q) &= \Delta_T^2(q_0)[1 + n_T(q)\ln(q/q_0) + O(n_T(q)^2)].\end{aligned}\quad (71)$$

Identifying the renormalization scale  $q_*$  with the pivot scale  $q_0$ , we then see that the spectral amplitudes given in (67) are correct to  $O(g_{\text{eff}}^4)$ , and that the corresponding spectral tilts are

$$n_S(q)-1 = -D_B g_{\text{eff}}^2 + O(g_{\text{eff}}^4), \quad n_T(q) = -D_A g_{\text{eff}}^2 + O(g_{\text{eff}}^4). \quad (72)$$

Comparing with the WMAP data, from Table 4 of [6] we find that  $(n_s-1) \sim O(10^{-2})$  at  $q = 0.002 \text{ Mpc}^{-1}$ , and hence  $g_{\text{eff}}^2 \sim O(10^{-2})$  also, justifying our perturbative treatment of the QFT.

To determine whether the spectral tilts are red or blue requires evaluating the signs of  $D_A$  and  $D_B$ , which will in general depend on the field content of the QFT. It is nonetheless still possible to extract predictions which are independent of the field content: for example, in these models, the scalar spectral index runs as

$$\alpha_s = dn_s/d\ln q = -(n_s-1) + O(g_{\text{eff}}^4). \quad (73)$$

This prediction is qualitatively different from slow-roll inflation, for which  $\alpha_s/(n_s-1)$  is of first-order in slow-roll [29], yet is nonetheless consistent with the WMAP observational constraints on  $n_s$  and  $\alpha_s$  given in [6] for a wide range of values of  $n_s$  and  $\alpha_s$ , as illustrated in Fig. 5.

#### 7.4. Non-Gaussianities.

Once  $N$ ,  $g_{\text{YM}}^2$  and the field content are fixed, all other cosmological observables (such as non-Gaussianities, *etc.*) follow uniquely from straightforward computations. We will present details of the correspondence between higher-order QFT correlation functions and non-Gaussian cosmological observables elsewhere [19]. Our results indicate, however, that the non-Gaussianity parameter  $f_{NL}^{\text{local}}$  [30] is independent of  $N$  to leading order, consistent with current observational evidence [6].

## 8. Conclusions.

Let us summarize the main results. We have presented a holographic framework for early-universe cosmology describing the period of time corresponding to the inflationary epoch. In particular, we have shown how to compute cosmological observables by performing calculations with a three-dimensional dual QFT. This procedure was discussed explicitly for the case of the primordial power spectrum, which is related to the 2-point functions of the dual QFT. Higher-point functions are related to non-Gaussianities, as will be discussed elsewhere. When gravity is weakly coupled at early times, holography correctly reproduces standard inflationary predictions for cosmological observables. When gravity is instead strongly coupled at early times, one finds new models that have a weakly coupled QFT description. We saw how models of this type exist that are compatible with current observations yet nevertheless have a distinct phenomenology from standard inflation. The proposed holographic approach thus provides a qualitatively new method for generating a nearly scale-invariant spectrum of primordial cosmological perturbations.

A special case to consider is de Sitter spacetime. A correspondence between de Sitter (dS) and CFT has been proposed in the past [31], and it is natural to wish to understand the relation of the present work to dS/CFT. Under the domain-wall/cosmology correspondence, dS is mapped to AdS, and our results for dS follow via suitable analytic continuation from the results for AdS. It is well known that quantum correlators, such as, for example, the Feynman propagators of massive scalar fields in AdS and dS spacetime, do not map to each other under analytic continuation, see for example [32]. This has been one of the obstacles in trying to establish a dS/CFT correspondence using analytic continuation. In our case, however, we do not map quantum de Sitter correlators to quantum AdS correlators. Instead, we map dS correlators directly to correlation functions of the dual QFT, and (as we have seen) one can successfully establish such a map. An approach to dS/CFT that has more in common with the present work is that of Maldacena [8], although our analytic continuation is different: we analytically continue both the momenta and Newton's constant, whereas in [8] it was the dS radius that was continued. Furthermore, compared to previous works [33, 34, 35, 36] that focused mostly on the computation of the scalar power spectrum for asymptotically dS geometries, we gave a complete discussion of both the scalar and tensor power spectra and our work includes the case of asymptotically power-law inflation. Finally, we proposed a precise definition of the dual QFT.

While uncovering an underlying holographic structure in inflationary cosmology is conceptually important, this in itself would lead to no new hard results were one to remain in the regime where inflationary computations are well-justified. The reason is that in this regime, the holographic computations simply reproduce well-known results. Rather, the power of the holographic approach is that it leads to new models in which the gravitational dynamics were strongly coupled at early times; for these models the standard inflationary computations do not apply. This is precisely the distinguishing feature of the present work: because we give an explicit definition of the dual QFT, we are able to obtain just such models with strongly coupled gravitational dynamics at early times. These models have a weakly coupled dual QFT description, permitting the analysis of a scenario that would otherwise be quite intractable.

We have seen how the dual QFT may be defined in an operational sense by first performing all computations with the ordinary QFT dual to a holographic RG flow, and then continuing the number of colors  $N$  and the momenta appropriately. In the large  $N$  limit the correlators are given as a series in  $1/N^2$ , and so the analytic continuation simply amounts to inserting minus signs. Furthermore, the continuation in momenta is such that the effective coupling  $g_{\text{eff}}^2 = g_{\text{YM}}^2 N/q$ , where  $q$  is a momentum, remains real. It would be very interesting to understand such 'pseudo'-QFTs from first principles. This would allow for a non-perturbative definition of the theory and would presumably elucidate many puzzling features of quantum gravity such as, for example, the entropy of de Sitter spacetime. Note also that the dual QFT provides a complete smooth

description of the system, including that of the initial singularity that is (generically) present in the background FRW solution. Our focus here was to extract the late-time behavior of the system, but it would be very interesting to understand the implications for singularity resolution.

For the present, however, the main question is whether or not holographic models exist that are compatible with current observations but have a distinct phenomenology from standard inflation. We have shown that the answer to this question is affirmative. Initiating a holographic phenomenological approach, we found that it is straightforward to satisfy the current observational constraints using simple QFT models containing only a few parameters. The near scale invariance of the cosmological power spectra follows immediately from simple dimensional considerations. A number of parameters in the QFT model may then be estimated or constrained using the present observational data; once the remaining parameters have been fixed, all cosmological observables (including non-Gaussianities) then follow from direct computation. Note that these are complete models: there are no UV issues as these theories are super-renormalizable and furthermore the dimensionful coupling constant acts as an infrared regulator. Even without knowing the values of all the parameters in the QFT model, it is still perfectly possible to obtain concrete predictions, since not all cosmological observables depend on the full set of QFT parameters. Examples include the scale invariance of the power spectra at leading order, as well as the running of the spectral index discussed above. In general, we expect to obtain predictions that are qualitatively different from those of standard inflationary scenarios based on weakly coupled gravity. These expectations are borne out by the form of the running we found for the scalar spectral index.

Clearly, the proposed phenomenological approach to holographic cosmology is worthy of further development. Over the next few years, forthcoming experiments (in particular the Planck satellite) promise to dramatically improve the observational constraints on many important cosmological parameters. It may well be that future observations confirm the predictions of holographic models of the type advocated here. The success of such an endeavor might then provide the first observational evidence for the holographic nature of our universe.

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### References

- [1] 't Hooft G 1993 Dimensional reduction in quantum gravity *Preprint* gr-qc/9310026; Susskind L 1995 The world as a hologram *J. Math. Phys.* **36** 6377 (*Preprint* hep-th/9409089)
- [2] Maldacena J M 1998 The large N limit of superconformal field theories and supergravity *Adv. Theor. Math. Phys.* **2** 231–52 (*Preprint* hep-th/9711200)
- [3] Gubser S S, Klebanov I R, and Polyakov A M 1998 Gauge theory correlators from noncritical string theory *Phys. Lett. B* **428** 105–14 (*Preprint* hep-th/9802109)
- [4] Witten E 1998 Anti-de Sitter space and holography *Adv. Theor. Math. Phys.* **2** 253–91 (*Preprint* hep-th/9802150)
- [5] McFadden P L and Skenderis K 2009 Holography for cosmology *Preprint* 0907.5542
- [6] Komatsu E *et al.* 2009 Five-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: cosmological interpretation *Astrophys. J. Suppl.* **180** 330 (*Preprint* 0803.0547)
- [7] Turok N 2002 A critical review of inflation *Class. Quant. Grav.* **19** 3449–67; Brandenberger R H 1999 Inflationary cosmology: progress and problems *Preprint* hep-ph/9910410
- [8] Maldacena J M 2003 Non-Gaussian features of primordial fluctuations in single field inflationary models *JHEP* **0205** 013 (*Preprint* astro-ph/0210603)
- [9] Cvetič M and Soleng H H 1995 Naked singularities in dilatonic domain-wall spacetimes *Phys. Rev. D* **51** 5768 (*Preprint* hep-th/9411170)
- [10] Skenderis K and Townsend P K 2006 Hidden supersymmetry of domain walls and cosmologies *Phys. Rev. Lett.* **96** 191301 (*Preprint* hep-th/0602260); 2007 Pseudo-supersymmetry and the domain-wall/cosmology correspondence *J. Phys. A* **40** 6733 (*Preprint* hep-th/0610253)

- [11] Bergshoeff E A, Hartong J, Ploegh A, Rosseel J and van den Bleeken D 2007 Pseudo-supersymmetry and a tale of alternate realities *JHEP* **0707** 067 (*Preprint* 0704.3559); Skenderis K, Townsend P K and van Proeyen A 2007 Domain-wall/cosmology correspondence in AdS/dS supergravity *JHEP* **0708** 036 (*Preprint* 0704.3918)
- [12] Skenderis K and van Rees B C 2008 Real-time gauge/gravity duality *Phys. Rev. Lett.* **101** 081601 (*Preprint* 0805.0150); 2009 Real-time gauge/gravity duality: prescription, renormalization and examples *JHEP* **0805** 085 (*Preprint* 0812.2909)
- [13] Salopek D S and Bond J R 1990 Nonlinear evolution of long wavelength metric fluctuations in inflationary models *Phys. Rev. D* **42** 3936
- [14] Bardeen J M, Steinhardt P J and Turner M S 1983 Spontaneous creation of almost scale-free density perturbations in an inflationary universe *Phys. Rev. D* **28** 679
- [15] Mukhanov V F, Feldman H A and Brandenberger R H 1992 Theory of cosmological perturbations *Phys. Rept.* **215** 203
- [16] DeWolfe O and Freedman D Z 2000 Notes on fluctuations and correlation functions in holographic renormalization group flows *Preprint* hep-th/0002226
- [17] Bianchi M, Freedman D Z and Skenderis K 2001 How to go with an RG flow *JHEP* **0108** 041 (*Preprint* hep-th/0105276)
- [18] Papadimitriou I and Skenderis K 2004 AdS/CFT correspondence and geometry *Preprint* hep-th/0404176; 2004 Correlation functions in holographic RG flows *JHEP* **0410** 075 (*Preprint* hep-th/0407071)
- [19] McFadden P L and Skenderis K to appear
- [20] Skenderis K 2002 Lecture notes on holographic renormalization *Class. Quant. Grav.* **19** 5849 (*Preprint* hep-th/0209067)
- [21] Kanitscheider I, Skenderis K and Taylor M 2008 Precision holography for non-conformal branes *JHEP* **0809** 094 (*Preprint* 0807.3324)
- [22] 't Hooft G 1974 A planar diagram theory for strong interactions *Nucl. Phys. B* **72** 461
- [23] Kanitscheider I and Skenderis K 2009 Universal hydrodynamics of non-conformal branes *JHEP* **0904** 062 (*Preprint* 0901.1487)
- [24] de Haro S, Solodukhin S N and Skenderis K 2001 Holographic reconstruction of spacetime and renormalization in the AdS/CFT correspondence *Commun. Math. Phys.* **217** 595 (*Preprint* hep-th/0002230)
- [25] Boonstra H J, Skenderis K and Townsend P K 1999 The domain wall/QFT correspondence *JHEP* **9801** 003 (*Preprint* hep-th/9807137)
- [26] Papadimitriou I and Skenderis K 2005 Thermodynamics of asymptotically locally AdS spacetimes *JHEP* **0508** 004 (*Preprint* hep-th/0505190)
- [27] <http://x-journals.com/2009/the-search-for-dark-energy-from-the-ground-up/>
- [28] Jackiw R and Templeton S 1981 How super-renormalizable theories cure their infrared divergences *Phys. Rev. D* **23** 2291; Appelquist T and Pisarski R D 1981 High-temperature Yang-Mills theories and three-dimensional quantum chromodynamics *Phys. Rev. D* **23** 2305
- [29] Kosowsky A and Turner M 1995 CBR anisotropy and the running of the scalar spectral index *Phys. Rev. D* **52** 1739 (*Preprint* astro-ph/9504071)
- [30] Komatsu E and Spergel D N 2001 Acoustic signatures in the primary microwave background bispectrum *Phys. Rev. D* **63** 063002 (*Preprint* astro-ph/0005036)
- [31] Strominger A 2001 The dS/CFT correspondence *JHEP* **0110** 034 (*Preprint* hep-th/0106113)
- [32] Das A K and Dunne G V 2006 Large-order perturbation theory and de Sitter/anti-de Sitter effective actions *Phys. Rev. D* **74** 044029 (*Preprint* hep-th/0607168)
- [33] Larsen F, van der Schaar J P and Leigh R G 2002 de Sitter holography and the cosmic microwave background *JHEP* **0204** 047 (*Preprint* hep-th/0202127)
- [34] Larsen F and McNeen R 2003 Inflation and de Sitter holography *JHEP* **0307** 051 (*Preprint* hep-th/0307026)
- [35] van der Schaar J P 2004 Inflationary perturbations from deformed CFT *JHEP* **0401** 070 (*Preprint* hep-th/0307271)
- [36] Larsen F and McNeen R 2004 Holography, diffeomorphisms, and scaling violations in the CMB *JHEP* **0407** 062 (*Preprint* hep-th/0402050)